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Physical auxiliary field in supersymmetric QCD with explicit supersymmetry breaking

Noriaki Kitazawa

*Department of Physics, Tokyo Metropolitan University,
Hachioji, Tokyo 192-0397, Japan*

Abstract

It is shown that the auxiliary field in the low-energy effective theory of the supersymmetric QCD ($SU(N_c)$ gauge symmetry with flavors $N_f < N_c$) can be understood as a physical degree of freedom, once the supersymmetry is explicitly broken. Although the vacuum expectation value of the auxiliary field is just a measure of the supersymmetry breaking in the perturbative treatment of the supersymmetry breaking, it can be the vacuum expectation value of the quark bilinear operator in the non-perturbative treatment of the supersymmetry breaking. We show that the vacuum expectation value remains finite in the limit of the infinite supersymmetry-breaking mass of the squark. We have to take the large N_c limit simultaneously to keep the low-energy effective Kähler potential being in good approximation.

1 Introduction

It is natural to attempt extracting some non-perturbative information on the non-supersymmetric gauge theory from the supersymmetric one by introducing the explicit supersymmetry breaking, since some non-perturbative effects can be reliably evaluated in the latter theory[1, 2, 3, 4, 5, 6, 7, 8]. In case of studying QCD using the supersymmetric QCD (SQCD)¹ there is a problem of physical degrees of freedom[9]. In the low-energy effective theory of SQCD there is no physical field which can correspond to the low-energy effective field in QCD. All the physical low-energy effective fields in SQCD describe the bound states which include at least one squark, and there is no effective field corresponding to the bound state whose constituents are quarks only. The auxiliary field in the effective “meson” superfield seems to correspond to the bound states of two quarks (mesons)[9], but it is not clear if the auxiliary field really can be a physical degree of freedom²[11].

In this note we show that the auxiliary component of the effective superfield in SQCD with $N_f < N_c$ can be understood as a physical degree of freedom. We introduce the supersymmetry-breaking squark mass in the ultraviolet theory, and construct its low-energy effective theory following the method in Refs.[12, 13, 14]. It gives the scalar potential which is described by two scalar fields, the first and auxiliary components of the effective superfield, and the vacuum expectation values (VEV) of these fields can be obtained. If we naively take the limit of the infinite squark mass, the VEV of the auxiliary field diverges. This result does not mean the unphysical nature of the field, but the bad approximation for the effective Kähler potential. If we take large N_c limit simultaneously, the VEV of the auxiliary field remains finite, and the auxiliary field can be understood as a physical degree of freedom corresponding to the meson in QCD. Large N_c limit is required to keep the effective Kähler potential in good approximation.

2 SQCD with explicit supersymmetry breaking

The Lagrangian of the supersymmetric $SU(N_c)$ gauge theory ($N_c > 2$) with flavors $N_f < N_c$ is

$$\mathcal{L} = - \int d^4\theta K + \left\{ \frac{1}{2} \text{tr} \int d^2\theta W^\alpha W_\alpha + \text{h.c.} \right\} \quad (1)$$

¹In this letter we use the word QCD to indicate general vector-like $SU(N_c)$ gauge theories with N_f flavors

²The low-energy effective scalar field with mass dimension three, which is the same mass dimension of the auxiliary field, is successfully utilized in Ref.[10].

and

$$K = Q_i^\dagger e^{-2gV} Q^i + \bar{Q}^{\dagger i} e^{2gV^T} \bar{Q}_i, \quad (2)$$

where Q^i and \bar{Q}_i are quark chiral superfields with flavor index i , V and W^α are the gluon vector superfield and its field strength chiral superfield, respectively, and g is the gauge coupling (see Ref.[15] for notation). The supersymmetry-breaking mass for squarks can be introduced as

$$\mathcal{L}_m = - \int d^4\theta X K, \quad (3)$$

where X is an interpolating vector superfield whose highest component has a VEV $\langle X \rangle = \theta^2 \bar{\theta}^2 m^2$. In the infinite mass limit the theory becomes non-supersymmetric QCD with an adjoint fermion.

It is known that the low-energy effective field of this theory with $m = 0$ is the gauge singlet chiral superfield

$$M^i_j \sim Q^i \bar{Q}_j. \quad (4)$$

The correspondence of its component fields to the composite operators in the fundamental theory is

$$A_M \sim A_Q A_{\bar{Q}} \quad \text{scalar component}, \quad (5)$$

$$\psi_M \sim \psi_Q A_{\bar{Q}} + A_Q \psi_{\bar{Q}} \quad \text{fermion component}, \quad (6)$$

$$F_M \sim -\psi_Q \psi_{\bar{Q}} \quad \text{auxiliary component}, \quad (7)$$

where the auxiliary fields of quarks are integrated out by using the equation of motion. Note that the auxiliary field F_M corresponds to the quark bilinear operator. The low-energy effective theory is described as

$$\mathcal{L}^{\text{eff}} = - \int d^4\theta K^{\text{eff}} + \left\{ \int d^2\theta W_{\text{dyn}} + \text{h.c.} \right\}, \quad (8)$$

where W_{dyn} is the dynamically generated superpotential

$$W_{\text{dyn}} = (N_c - N_f) \left(\frac{\Lambda^{3N_c - N_f}}{\det M} \right)^{\frac{1}{N_c - N_f}} \quad (9)$$

and Λ is the scale of dynamics[12]. It is very difficult to obtain the exact effective Kähler potential. We use

$$K^{\text{eff}} = 2\text{tr} \sqrt{M^\dagger M} \quad (10)$$

which is effective in the weak coupling limit and describes the theory on the classical flat direction [13, 14].

The supersymmetry breaking effect due to X can be introduced to the effective theory by including the terms which are the general functions of X and M . Since X is a vector superfield and we can consider as $X^2 = 0$, only the Kähler potential has correction of the form

$$K_m^{\text{eff}} = XF(M^\dagger, M) \quad (11)$$

in the low-energy limit, where F is a real function. We estimate the function F as follows.

Since the Kähler potential of Eq.(10) is obtained by a simple replacement of Eq.(2) based on the identification of Eq.(4), we may be able to do the same replacement in Eq.(3). But the replacement itself must be modified by the supersymmetry breaking effect. From the condition of the flat direction, we have[13]

$$(\bar{Q}^\dagger \bar{Q})^i_k (\bar{Q}^\dagger \bar{Q})^k_j = (\bar{Q}^\dagger Q^\dagger)^i_k (Q \bar{Q})^k_j. \quad (12)$$

In the supersymmetric case the right hand side is simply replaced by $M^\dagger M$, but now we should have

$$(\bar{Q}^\dagger \bar{Q})^i_k (\bar{Q}^\dagger \bar{Q})^k_j = M^{\dagger i}_k M^k_j + XG(M^\dagger, M)^i_j, \quad (13)$$

where G is a real function of M^\dagger and M . This relation results

$$\text{tr}(\bar{Q}^\dagger \bar{Q}) = \text{tr} \sqrt{M^\dagger M} + \frac{1}{2} X \text{tr} \left((M^\dagger M)^{-\frac{1}{2}} G(M^\dagger, M) \right). \quad (14)$$

Therefore, we have

$$F(M^\dagger, M) = 2\text{tr} \sqrt{M^\dagger M} + \text{tr} \left((M^\dagger M)^{-\frac{1}{2}} G(M^\dagger, M) \right) \quad (15)$$

in the weak coupling limit. The result of this note indicates that the first term dominates the second term in large N_c limit. Here, we simply neglect the second term. The effective theory which we are going to analyze is

$$\mathcal{L} = - \int d^4\theta (1 + X) K^{\text{eff}} + \left\{ \int d^2\theta W_{\text{dyn}} + \text{h.c.} \right\}. \quad (16)$$

We can obtain the effective scalar potential from this Lagrangian. For simplicity, we assume that the VEV of the effective field is flavor diagonal:

$M^i_j = \Phi \cdot \mathbf{1}^i_j$. The scalar potential is described by two scalar fields, A_Φ and F_Φ , the scalar and auxiliary components of the superfield Φ , respectively.

$$V = -\frac{N_f}{2} \frac{F_\Phi^\dagger F_\Phi}{\sqrt{A_\Phi^\dagger A_\Phi}} + 2N_f m^2 \sqrt{A_\Phi^\dagger A_\Phi} + \left\{ N_f \frac{F_\Phi}{A_\Phi} \left(\frac{\Lambda^{3N_c - N_f}}{A_\Phi^{N_f}} \right)^{\frac{1}{N_c - N_f}} + \text{h.c.} \right\}. \quad (17)$$

If we perturbatively solve the stationary conditions, $\partial V / \partial A_\Phi = 0$ and $\partial V / \partial F_\Phi = 0$, the VEV of the auxiliary field is simply proportional to m^2 , and it is merely a measure of the supersymmetry breaking (some regulator interaction, $W_{\text{reg}} = \det M / \mu^{2N_f - 3}$, for example, must be introduced to have a finite VEV of A_Φ in the supersymmetric limit). We can easily solve the stationary conditions without any approximation. The result is

$$A_\Phi = \left(\frac{\Lambda^{3N_c - N_f}}{(Cm)^{N_c - N_f}} \right)^{\frac{1}{N_c}}, \quad (18)$$

$$F_\Phi = 2C^{\frac{N_f}{N_c}} \left(m^{N_f} \Lambda^{3N_c - N_f} \right)^{\frac{1}{N_c}}, \quad (19)$$

where $C \equiv \sqrt{(N_c - N_f)/(N_c + N_f)}$.

3 Decoupling limit and VEV of the auxiliary field

Before taking the limit of $m \rightarrow \infty$, the scale of dynamics at low energies, Λ' , must be specified. Since we take m very large in comparison with any other scales of dynamics, the matching condition of the running gauge coupling at one-loop can be utilized. The scale Λ can be described by the scale Λ' and m as

$$\Lambda = \Lambda' \left(\frac{\Lambda'}{m} \right)^{\frac{b' - b}{b}}, \quad (20)$$

where $b \equiv 3N_c - N_f$ and $b' \equiv 3N_c - \frac{2}{3}N_f$. Then, we have

$$A_\Phi = C^{-\frac{N_c - N_f}{N_c}} (\Lambda')^2 \left(\frac{m}{\Lambda'} \right)^{-1 + \frac{2N_f}{3N_c}}, \quad (21)$$

$$F_\Phi = 2C^{\frac{N_f}{N_c}} (\Lambda')^3 \left(\frac{m}{\Lambda'} \right)^{\frac{2N_f}{3N_c}}. \quad (22)$$

In $m \rightarrow \infty$ limit with fixed Λ' , F_Φ diverges, though A_Φ vanishes as expected. This is because of a truncation in the effective Kähler potential in the previous section. We show that we can obtain a convincing result in large N_c

limit, which indicates that the first term in Eq.(15) dominates the second term in large N_c limit.

To take a correct large N_c limit the scales of dynamics, Λ and Λ' , should not depend on N_c . This requires that

$$\frac{\Lambda'}{\Lambda} = \left(\frac{m}{\Lambda'}\right)^{\frac{b'-b}{b}} \rightarrow \left(\frac{m}{\Lambda'}\right)^{\frac{N_f}{9N_c}} \quad (23)$$

should be a finite value. If we take the finite value as $\beta^{1/6}$ ($\beta > 1$ because of $\Lambda' > \Lambda$), we have

$$A_\Phi \rightarrow (\Lambda')^2 \beta \beta^{-\frac{3N_c}{2N_f}} \rightarrow 0, \quad (24)$$

$$F_\Phi \rightarrow 2 (\Lambda')^3 \beta \quad (25)$$

in large m and N_c limit. The uncertainty of the value of β corresponds to the ambiguity in the present determination of the scale of dynamics at low energies. Now we have a finite value of F_Φ and vanishing A_Φ which is required to have non-decoupling quarks. This result shows that the auxiliary field can be understood as a physical degree of freedom.

We note that the Kähler potential of Eq.(10) is in good approximation, since the value of the gauge coupling in the supersymmetric theory at the scale Λ' which is the typical scale of the low-energy physics is small.

$$\alpha(\Lambda') = \frac{2\pi}{(3N_c - N_f) \ln(\Lambda'/\Lambda)} \rightarrow \frac{2\pi}{3N_c \ln \beta^{1/6}} \quad (26)$$

We also note that F_Φ diverges in this large m and N_c limit, if we assume the naive effective Kähler potential $K_{\text{naive}}^{\text{eff}} \propto \text{tr}(M^\dagger M)/\Lambda^2$.

The next question is which degree of freedom in the underlying theory corresponds to the auxiliary field. As described in Eq.(7), it is natural to consider that the quark bilinear operator corresponds to the auxiliary field, though the large N_c scaling is different ($\langle \psi_Q \psi_{\bar{Q}} \rangle \sim N_c$). This means that the coefficient which should be determined by the dynamics depends on N_c . Namely,

$$F_M^{ij} \propto -\frac{1}{N_c} \psi_Q^i \psi_{\bar{Q}j} \propto \frac{1}{\sqrt{N_c}} \Sigma^i_j, \quad (27)$$

where Σ is the complex scalar field in the linear σ -model as the low-energy effective theory of QCD with an adjoint fermion.

4 Conclusion

It has been shown that the auxiliary component of the low-energy effective chiral superfield in the supersymmetric QCD ($N_f < N_c$) with explicit supersymmetry breaking can be understood as a physical degree of freedom. Supersymmetry was explicitly broken by the squark mass m , and the large m limit was considered to obtain the non-supersymmetric theory: QCD with an adjoint fermion. Large N_c limit was required to keep the effective Kähler potential in good approximation. The finite vacuum expectation value of the auxiliary field was obtained in the limit, which means that the auxiliary field can be a physical degree of freedom in the non-supersymmetric theory. It can be identified to the quark bilinear operator with a coefficient of $1/N_c$ scaling or the meson field with a coefficient of $1/\sqrt{N_c}$ scaling.

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